Example 7: Consider the set of vectors span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$) where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{4}$$

1. Show that the vector $\mathbf{v}_3$ is in span($\mathbf{v}_1, \mathbf{v}_2$).

Since $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$, $\vec{v}_3$ is a linear combination of $\vec{v}_1, \vec{v}_2$ and thus $\vec{v}_3$ is in span$(\vec{v}_1, \vec{v}_2)$.

2. Show that any vector in span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$) is also in span($\mathbf{v}_1, \mathbf{v}_2$).

Suppose $\vec{v}$ in span$(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

Then $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

$= c_1\vec{v}_1 + c_2\vec{v}_2 + c_3(\vec{v}_1 + \vec{v}_2)$

$= (c_1 + c_3)\vec{v}_1 + (c_2 + c_3)\vec{v}_2$. Thus, $\vec{v}$ in span$(\vec{v}_1, \vec{v}_2)$.

3. Show that any vector in span($\mathbf{v}_1, \mathbf{v}_2$) is also in span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$).

Suppose $\vec{v}$ in span$(\vec{v}_1, \vec{v}_2)$.

Then $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2$

$= c_1\vec{v}_1 + c_2\vec{v}_2 + 0\,\vec{v}_3$. Thus $\vec{v}$ in span$(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

4. Thus span($\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$) is equal to span($\mathbf{v}_1, \mathbf{v}_2$) and hence is a Plane in $\mathbb{R}^3$.

> *Theorem 3:* Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ in $\mathbb{R}^m$. If $\mathbf{v}_i$ is in span($\mathbf{v}_1, \ldots, \mathbf{v}_{i-1}, \mathbf{v}_{i+1}, \ldots, \mathbf{v}_n$), then
>
> $$\text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_n) = \text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_{i-1}, \mathbf{v}_{i+1}, \ldots, \mathbf{v}_n)$$

$\vec{v}_i$ a "redundant" vector